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MODELING INNOVATION DIFFUSION

C. MARCHETTI
International Institute for
Applied Systems Analysis
Laxenburg, Austria

1. The Methodology

The problems of market dynamics for primary energies led to a reassessment and a fresh look at the concepts developed by Lotka and Volterra in the twenties, in relation to the dynamics of ecological systems. That reassessment led to a fresh view of the working of social and economic systems, reducing it to single mechanisms and single mathematics, although the wheels in wheels of the actual system make it appear extremely complex.

The basic mechanism is *diffusion* at a cultural level. A process of transmission and verification of cultural packets that set paradigms for personal and social action. Diffusion processes can be monitored by field measurements and I refer to Hägerstrand of the University of Lund, Sweden, as the precursor and guru of basic research on (technological) diffusion in social systems. In a current diffusion process the rate of adopters (adopters per unit of time) is represented in Figure 1. One can also be interested in the cumulative number of adopters and the resulting curve is reported in Figure 2.

As shown in the appendix this curve results from a single epidemic differential equation where people falling sick are proportional to the number of *sick people* (the carriers), multiplied by the number of those *susceptible.* The sick grow exponentially until the susceptible group starts to thin out, making transmission more and more difficult.

The equation representing the function of Figure 2 is a logistic equation. In most of the charts I will use a transform that linearizes the S-curve manipulating the ordinates (Figure 3). It is called the Fisher–Pry transform, where F are the actual data measured as a fraction of the final saturation level. The ordinates report log F/1–F. The saturation level value has then to be provided separately, and is the number in parentheses. The rate constant of the equation is visualized through a time span, i.e., to go from F = 0.1 to F = 0.9, covering then 80% of the process.

2. Area of Application

We applied this technique of analysis to the dynamics of energy markets and energy uses, from the world to single industry, using about 300 cases where the model showed its unequaled capacity to describe consistently the dynamics of situations over incredibly large

B. Henry (ed.), Forecasting Technological Innovation, 55–77.

spans of time. We came slowly to the realization that appropriate measures of social and economic systems show an ***inherent stability in their evolution*** and that the model applies at all hierarchical levels showing the basic ***quasifractal nature of the system.*** The quasi comes from the fact that the equations are always the same, but not their parameters.

In order to complete this methodological introduction I should say that diffusion can be multiple, in the sense that in the same niche we can have various competitors diffusing at the same time, *in* or *out.* Multiple diffusion is of utmost importance to correctly interpret the evolution of the system, and its neglect was the main cause of failure of preceding attempts to map systems with this model. The last point is that the cases analyzed amount to about 3000, zooming into all imaginable areas.

3. The Case of Innovations

Here I shall concentrate on the problem of innovation diffusion, touching however the boundary areas that give meaning to this diffusion. In particular, looking upstream to the process of R&D, invention, innovation, and entrepreneurship that bring innovations to the markets. And downstream or better contextually, looking at the Kondratieff long economic cycles that quintessentially regulate the diffusion of the innovations into the markets.

How can an innovation be defined? I think the clearest definition is that of Mensch: An innovation is an invention entering the market. This definition is of obvious application for material products. But many innovations are immaterial, e.g., new schemes of organization or new financial and banking tools. In this case, the start of the adoption, i.e., of use by banks and financial institutions, is the signal that the invention is becoming an innovation.

3.1. THE EXAMPLES

I will now jump *in medias res* by giving some examples of diffusion processes to fix the ideas and show how precisely they follow the model. First, as I said, the paradigm is biological and an epidemic has the highest visibility. Let us take one of the London plagues (Figure 4) monitored through the cumulative number of deaths. This is really a proxy, but deaths are usually counted very carefully which makes the statistics quite reliable. Total deaths were 54,700, of which 80% occurred in a period of 8.5 months, around the point where the rate of death was maximum, May 1665.

Biological systems are quintessentially informational systems, and a plague can be seen as the consequence of a computer virus destroying the software of a class of computers. Pure star wars. In this way a message diffusion in a close social system, like a boys' camp, could have the same course. The experiment has been made and the results, elaborated through our model, are reported in Figure 5. The diffusion, as monitored by the psychologists doing the experiment, followed precisely an epidemic course. However, there is an interesting detail: the saturation point is 37 when the number of boys in the camp was 42. As in the case of the plague, part of the population is "informationally inaccessible". This provides the opportunity for introducing the concept of niche, so important in biology and in ecology, as the (abstract) territory over which a message can diffuse. This saturation point can be calculated by best fitting the actual diffusion data with a logistic or a more complex system of equations into the niche, depending on the number of competing messages. See the appendix for a discussion on "best fit".

An innovation is not necessarily a new machine, it is basically a new way of doing something, old or new. Postage stamps were introduced to simplify the payment of the fee to the post office. The innovation took place in Britain in 1840 and diffused the world over. The niche is limited to Europe and South America following the statistical basis taken from a paper by Pemberton (Figure 6). It is interesting to note that in spite of the simplicity of the system the idea was not adopted instantly. It took 26 years to catch 80% of the adopters. These 26 years, or numbers around that value, appear as time constants for the adoption of innovations at the social level. Social time appears in fact to be divided into time boxes of about 55 years where diffusion processes begin and end. The same object can start a new diffusion in the next box with a higher (or lower!!) saturation point. These boxes can be identified with Kondratieff's long economic cycles. One of them goes from 1830 to 1885 and we see that our stamp adoption process is precisely encapsuled in it.

A democracy is one way of organizing political life, and the definition is not sharp as in the case of postage stamps. We need an expert to tell the difference between a good and a bad one. As an example I used the work of a famous historian, Professor Modelsky, who counted democratic states since 1800, the beginning of democracies in the modern sense. Since the number is relatively limited, a new member in or an old out, generates perceptible deviations. In spite of that, the general trend (see Figure 7) follows the model quite closely. The time constant here is 150 years. Adopting democracy is more complicated than adopting stamps. The full process takes 300 years. The limited value of the saturation level should not induce excessive pessimism. There is a very long-term tendency in political coalescence. When Europe has a single political head, twelve democratic states will disappear. A neat question could be how the model knows that. This is one of the magic aspects – the system seems to know that. Distant phenomena have very deep roots and, in between, a very selfconsistent deployment.

Reorganizing the political system around the paradigm of democracy is a major enterprise and requires a lot of time and tinkering. But also subsystems can be organized and reorganized and the process should have a similar structure even if there are shorter time constants. After conquering Central–South America, Spain organized its mining districts to produce a flow of precious metals. The flow of silver (cumulative) is reported in Figure 8. Here I have intentionally introduced the concept of innovation diffusion, somehow at the margin of its current use in order to show its potential when innovative points of view are used.

I would not, however, like to deceive those who think that innovation diffusion has to be seen basically as the diffusion of innovative products. The population of elementary robots in Japan is reported in Figure 9 and that of more sophisticated robots in Figure 10. The time constants are here very short but in both cases saturation occurs in the 1990s. As we shall see in other examples, this is the effect of the Kondratieff wall. Because diffusion mostly saturates at the wall, when diffusion begins near the wall the diffusion process is very steep.

Another case of gadgets nearer to the consumer level is that of automobiles. For a detailed discussion see my contribution "Technological innovation in transport" in this issue. Here I use the isolated example of car penetration in Italy. The number of registered cars is reported in Figure 11, i.e., the cars on the road. The time constant is 22 years, as for the case of tractors in France reported in Figure 12. Also here we see clearly the Kondratieff wall effect. At the end of the diffusion wave the number may exceed the saturation level and also oscillate around it. This is a complex phenomenon and will not be discussed further here. It occurs also for biological systems. On the one hand, it can be interpreted as "hunting" the asymptote in order to search for its actual value. Or, on the other hand, the proxies we use for the statistical basis are not completely appropriate. Cars are a means of basic personal transport. But they can become collectors' items or old pieces of machinery parked in the second house and only occasionally used. They take different niches and should be counted separately.

3.2. ENTREPRENEURSHIP

I will now cast my net on the process that precedes the introduction of innovations into the market: R&D and entrepreneurship. Current wisdom says that these three things are essentially stochastic, linked to many microcontexts whose complication makes any precise analysis an improbable success. However, in the twenties in Vienna, Schumpeter was preaching about the existence of opportunity windows, at least for entrepreneurship, linked to the general pulsations in the economy. In his view, entrepreneurs, by launching new products during the recession periods, open new lines of activity helping the system to recover into a boom period. About 20 years ago, Mensch counted basic inventions and innovations using selfconsistent definitions that make his statistical findings a good starting point for further processing (Figure 13).

I did this in 1973, and discovered that Mensch's waves in inventions (making the idea work) and innovations (selling the product) are highly organized, both internally, and as a group. The first cumulative numbers of inventions and innovations is reported in Figure 14, fitted with appropriate logistics. The excellent fit supports my subjacent idea that both occur when there is a demand within the system. The demand diffuses, creating an offer with that particular shape.

3.3. BASIC INNOVATION WAVES

Innovation waves more in focus with our interests are synthetically reported in Figure 15. Three of them result from fitting Mensch's data; the other two are calculated using the internal consistencies of the set. One of these consistencies lies in the fact that the distance between the center points of the innovation waves is always 54 years, the length of a Kondratieff cycle. Another consistency is that a new primary energy is associated with each wave. The lines in Figure 15 marked with the names of a primary energy actually represent its market diffusion. The innovation wave marked 8 is calculated, but nuclear "popping up" at its root is actual.

For details of mapping and forecasting inventions and innovation waves see Marchetti (1972), where the present wave of innovations was forecast to start at the beginning of the eighties, peaking in 1993. The wave is certainly here and still surging. In a few years, we may check the beginning of the ebb. For the next wave marked 10 in Figure 15, the associated new primary energy should start its penetration around 2025. There is a curious coincidence that most forecasts about the introduction of fusion energy based on detailed planning converge around that date.

3.4. KONDRATIEFF CYCLES

Another way of looking at the Kondratieff cycles has been proposed by Stewart of Nutevco. He found that best fitting total energy consumption or electricity consumption in the USA with exponential or logistic curves omitted some regular deviations. By separating them in terms of percentage deviation from the best fit he found that they match fairly well a sinusoid with a period of 55 years and an amplitude of ±20% (Figure 16). Energy and electricity consumption represent a good aggregate measure of societal metabolism, which obviously pulsates with a period of 55 years. To locate invention and innovation waves into this pulsation I reported the central points of the invention waves in square boxes and these of innovation waves in round ones. The upward and downward movements of energy consumption relative to the long-term trends represent boom and recession periods respectively. The center of innovation waves seems to be locked into the bottom of recessions, precisely in the spirit of Schumpeter.

As Schumpeter said, entrepreneurs develop the new industries that bring the system out of the doldrums and into a new boom activity. The process is not painless. I analyzed the opening of plants making cars in the USA and Europe. US statistics not only gave the dates of opening (selling the first car) for the various companies that "popped up" in the first part of this century, but also the dates when they were finally closed.

The diffusion wave of entrepreneurs entering the game is reported in Figure 17, fitted with a logistic, together with the wave of the quitters. A reflection on the results of Figure 17 is slightly depressing. The saturation point for the pioneers is 1400. This is the same for the quitters. The survivors remain hidden in the folds of the inevitable imprecisions in the estimates of the saturation point. These are in fact a dozen, less that 1%. The two lines are remarkably parallel showing a mean lifetime for these new companies of about 4 years. Other cases I have examined show this is not an isolated case. The carnage of the entrepreneurs is actually the rule.

A modern hint in that direction is given by the cumulative number of companies which entered the market producing mainframe computers (Figure 18). They appear to saturate around 700. If we look at new models put on the market (Figure 19), we find a saturation point around 3000. That makes about 4 models per manufacturer. The same analysis for the big ten (Figure 20) shows that together they will produce about 800 new models. The others will be left with about three models each, a sure guarantee that they will not survive for long.

At this point I think the main line of attack, for the problems of invention, innovations, and their diffusion, is set. Our very simple tool of analysis has shown very deep and long-lasting regularities in the system. They in fact encompass almost any fold we were able to quantify in time. The number of cases examined at IIASA is about 3000. We have gathered enough experience to avoid the pitfall and use these regularities and stabilities to efficiently forecast.

3.5. THE SPREAD OF A PRODUCT

I end with an interesting example. Ethylene production in Europe up to 1972 is reported in Figure 21 together with the forecast of the European Association of Chemical Industry (CEFIC). Our best fit is reported in Figure 22, together with a safety net for the almost inevitable oscillations around the saturation point. The situation in 1977 is reported in Figure 23 together with the revised forecast of CEFIC. But optimism does not always pay. A new snapshot is given in Figure 24, including the second revision to the CEFIC forecast which is regularly contradicted by the situation up to 1987 (Figure 25). The only sad point is that not taking account of these processes has cost perhaps a trillion dollars to world industry in overcapacity.

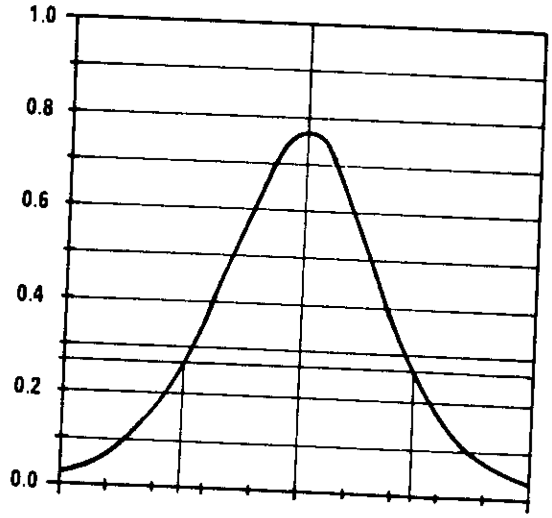

Figure 1. The pulse.

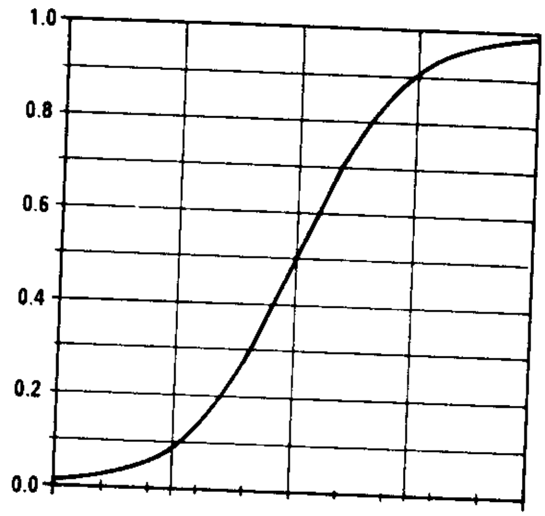

Figure 2. Logistic function as integral of the pulse.

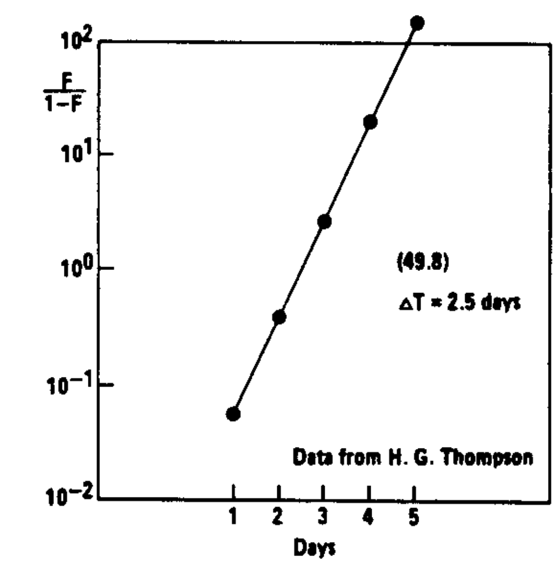

Figure 3. Growth of bacterial colony (cm^2). Source: Marchetti (1986).

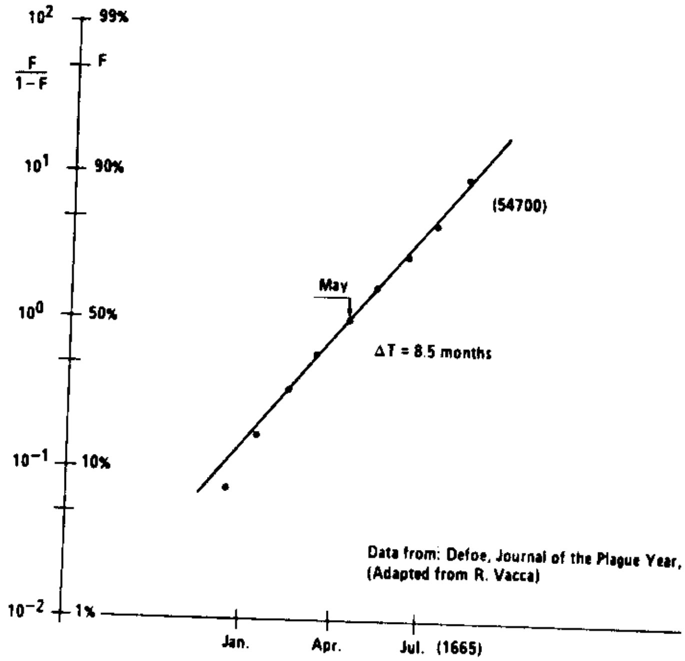

Figure 4. Cumulative deaths (officially counted) during London plague of 1665. Source: Marchetti (1987).

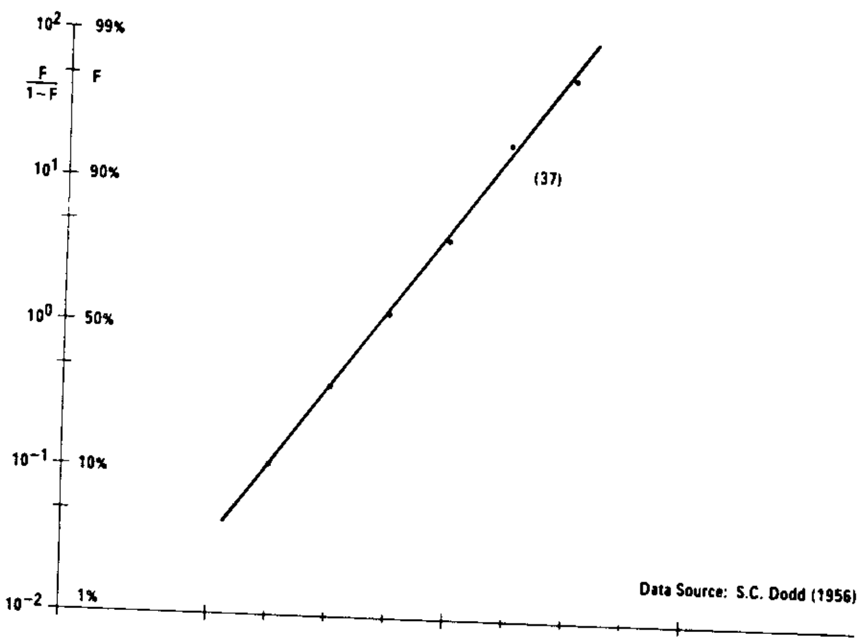

Figure 5. Diffusion of message in boys camp (population of 42 boys). Source: Marchetti (1988).

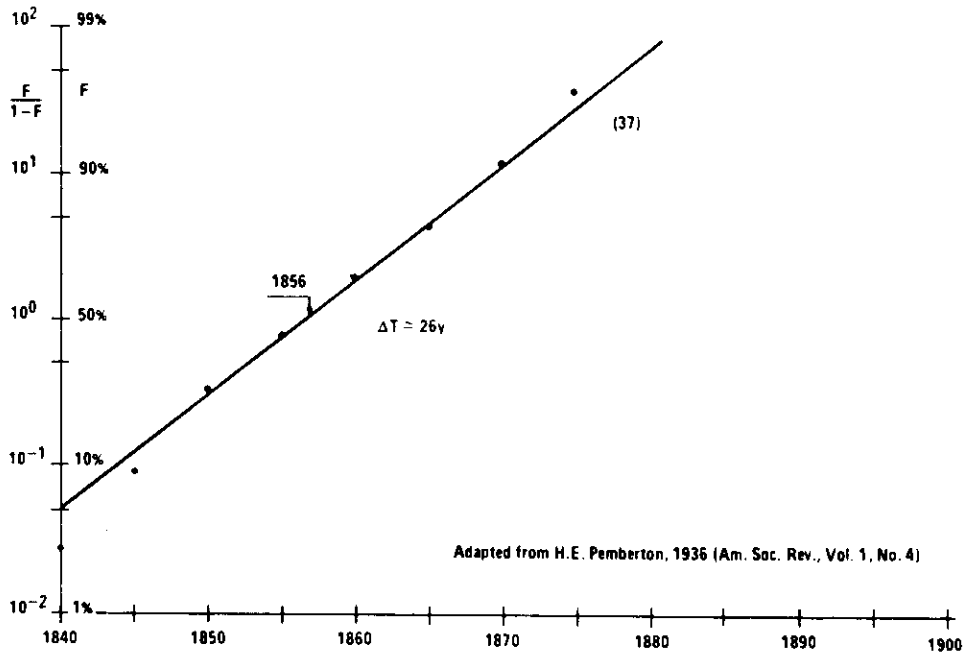

Figure 6. First adoption of postage stamp by European and South American nations. Source: Marchetti (1987).

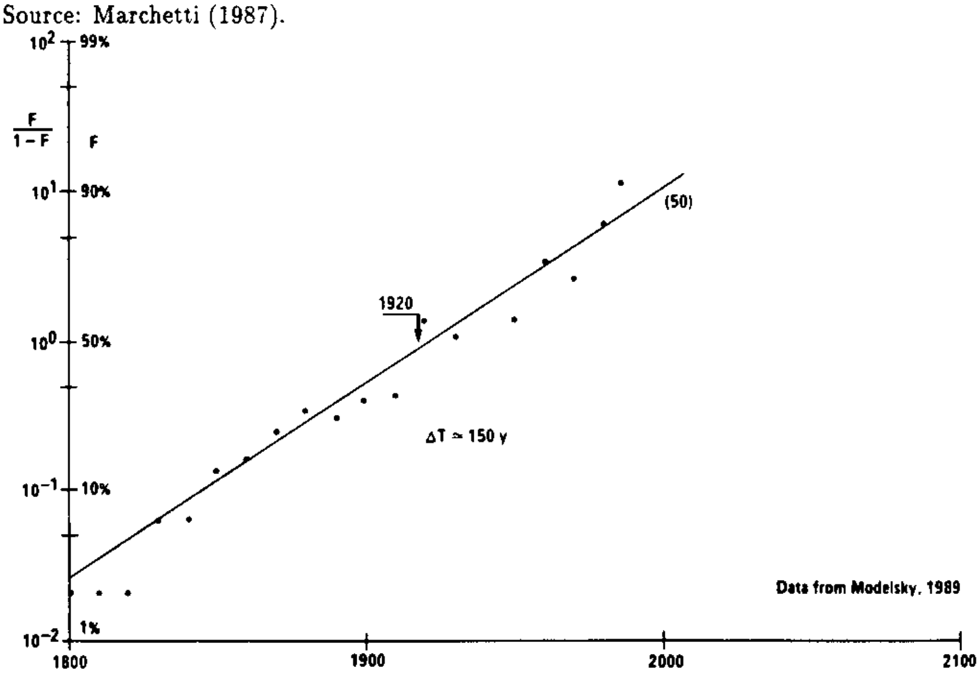

Figure 7. Number of democratic states. Source: Marchetti (1989).

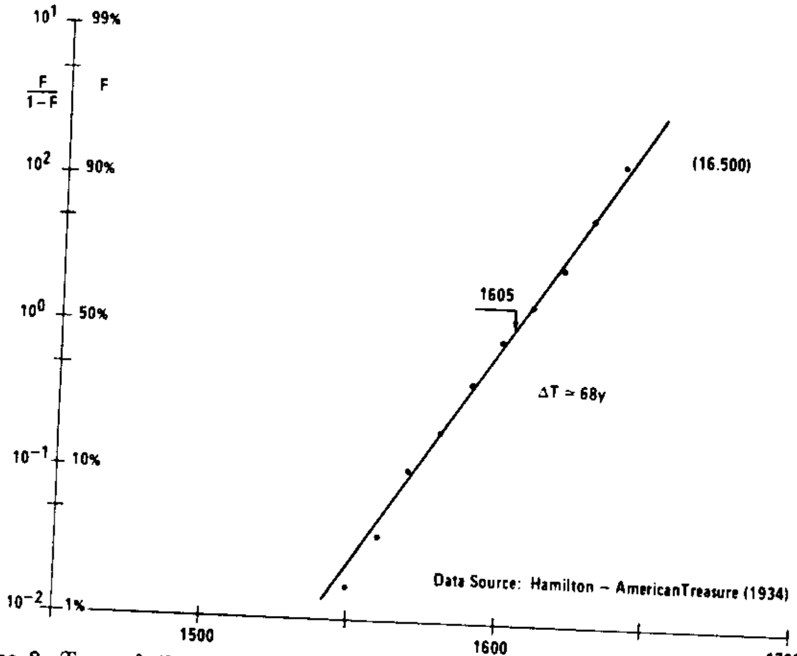

Figure 8. Tons of silver imported from America to Spain. Source: Marchetti (1988).

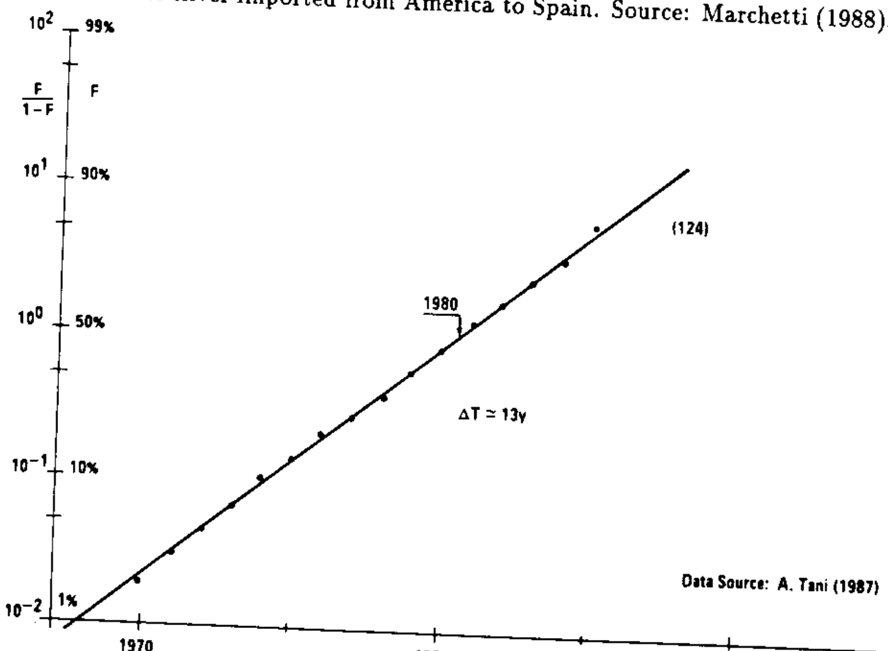

Figure 9. Elementary robots in Japan: Manual manipulation, fixed and variable sequence (in thousands). Source: Marchetti (1988).

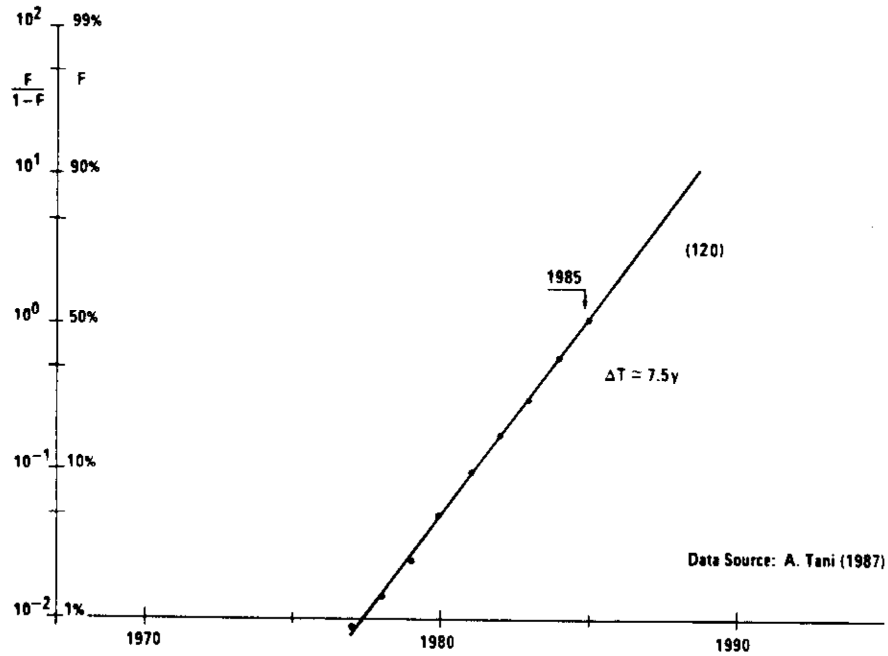

Figure 10. Advanced robots in Japan: Playback – NC and intelligent robots (in thousands). Source: Marchetti (1988).

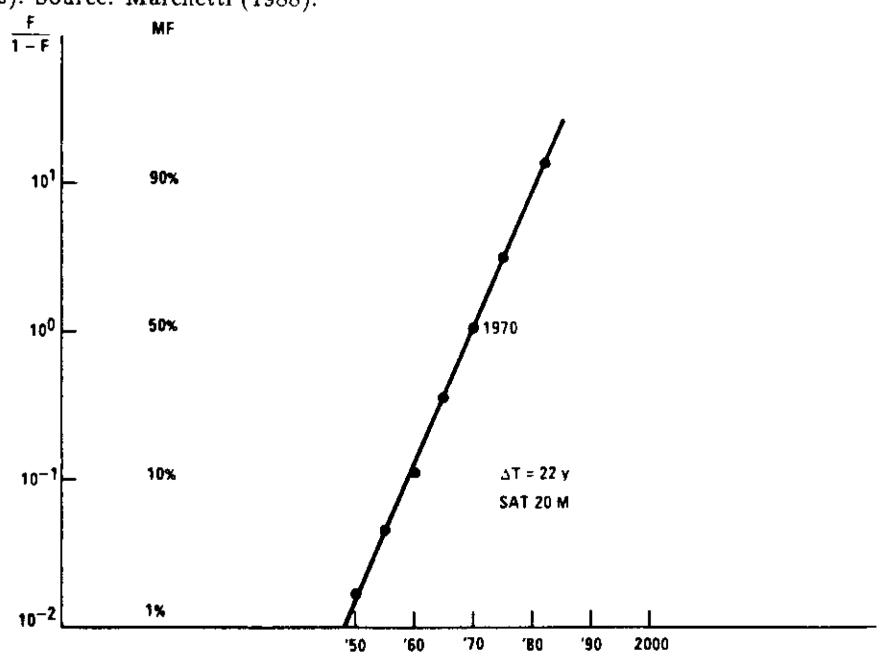

Figure 11. Car registration in Italy.

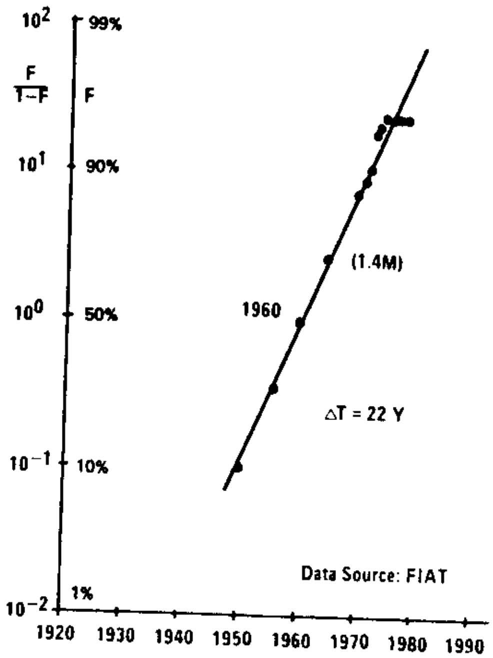

Figure 12. Tractor population in France: 95% saturation in 1980. Source: Marchetti (1982).

	Invention	Innovation
Electricity production	1708	1800
Blast furnace with coke	1713	1796
Photography	1727	1838
Crucible steel	1740	1811
Lead-chamber process	1740	1819
Insulated conductor	1744	1820
Portland cement	1756	1824
Locomotive	1769	1824
Pharmaceutical production	1771	1827
Pulled wire	1773	1820
Rolled rails	1773	1835
Potassium chlorate	1777	1831
Puddling furnace	1783	1824
Quinine production	1790	1820
Telegraph	1793	1833
Arc lamp	1810	1844
Pedal bicycle	1818	1839
High-voltage generator	1820	1849
Electric-impulse stimulator	1831	1846
Vulcanized rubber	1832	1852
Deep-sea cable	1847	1866

Figure 13. The 1802 cycle. Source: Mensch (1975).

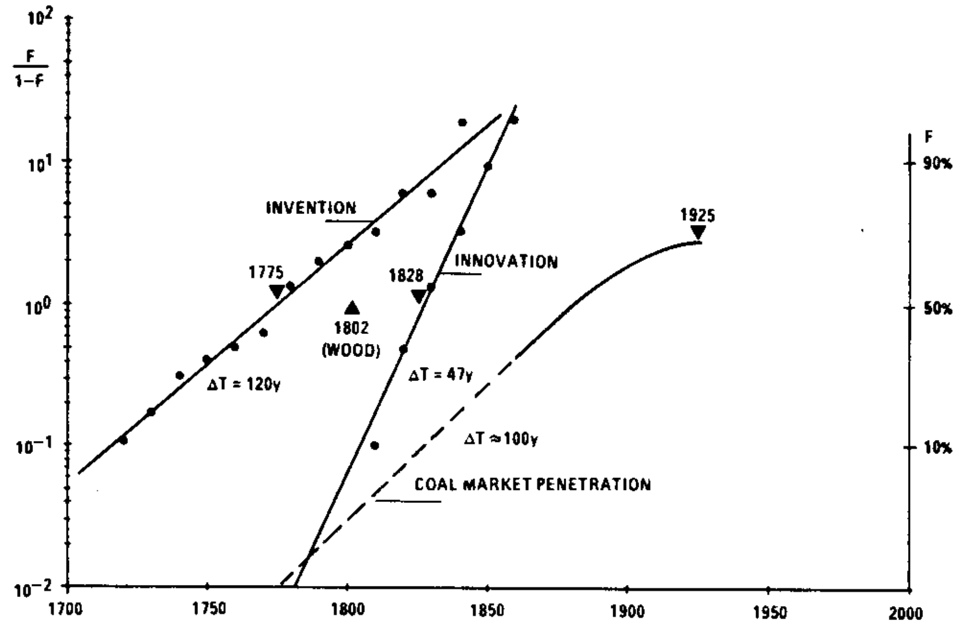

Figure 14. The 1802 wave.

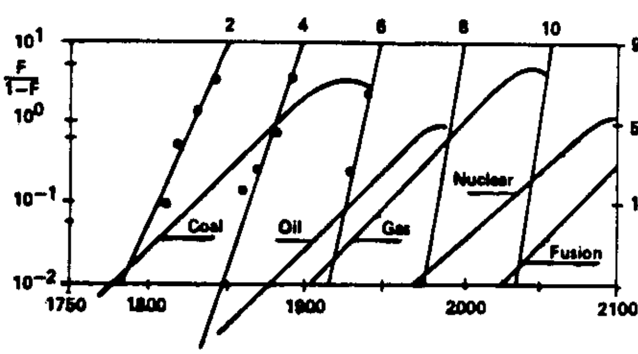

Figure 15. Innovation waves and the start of new energy sources.

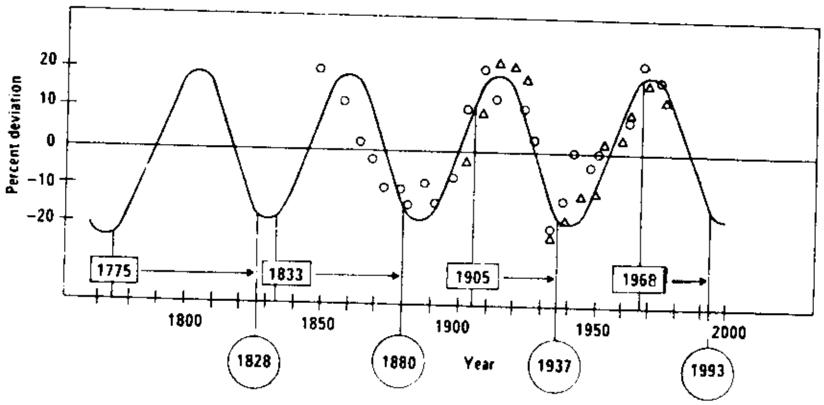

Figure 16. Center of invention and innovation waves located on energy indicator.

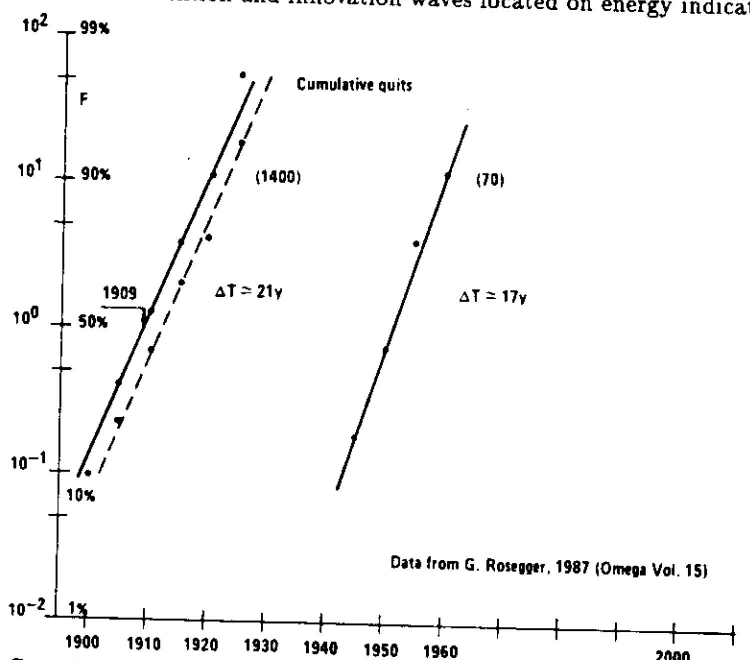

Figure 17. Cumulative new entries of car makes in the USA. Source: Marchetti (1987).

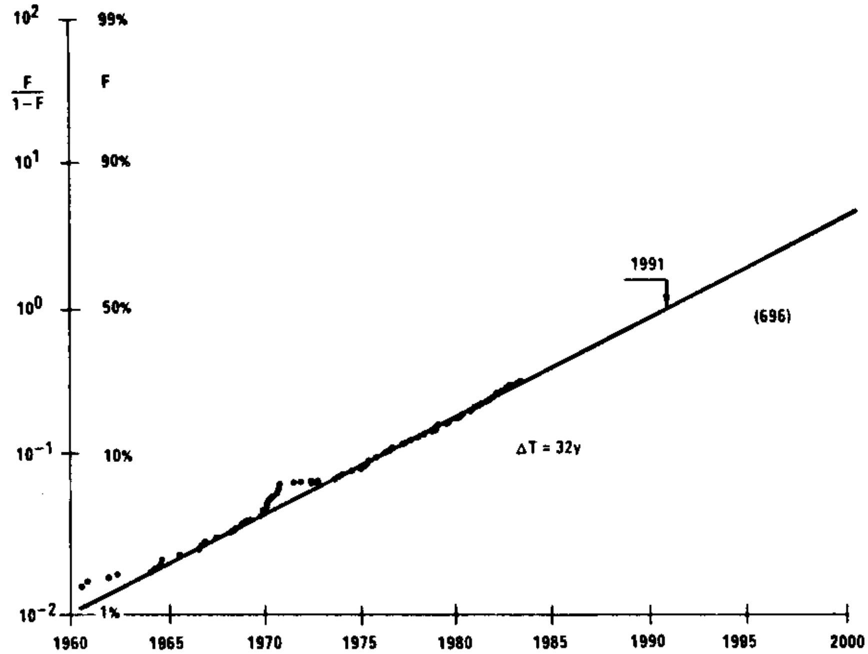

Figure 18. New computer manufacturers population.

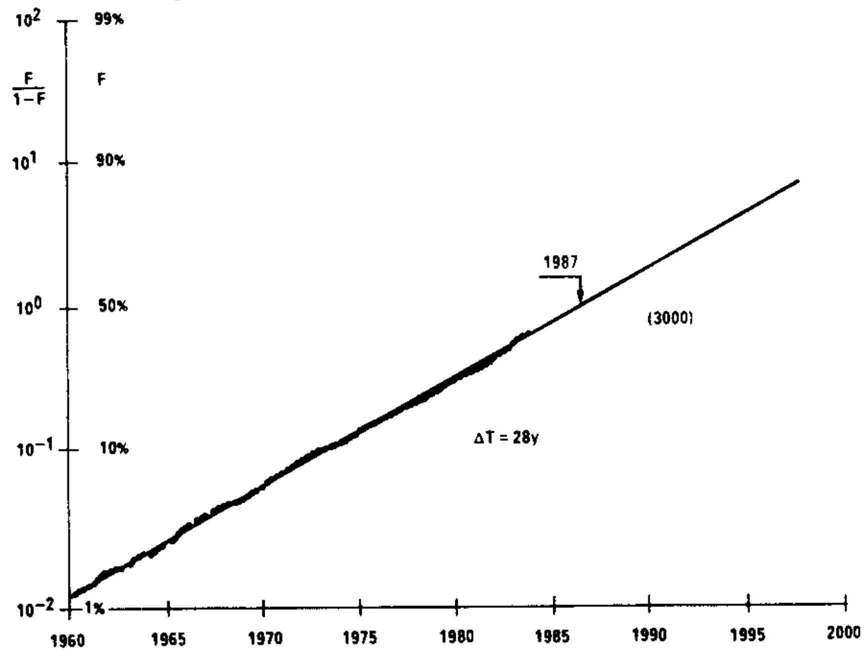

Figure 19. Innovation in computer industry: New models, all manufacturers.

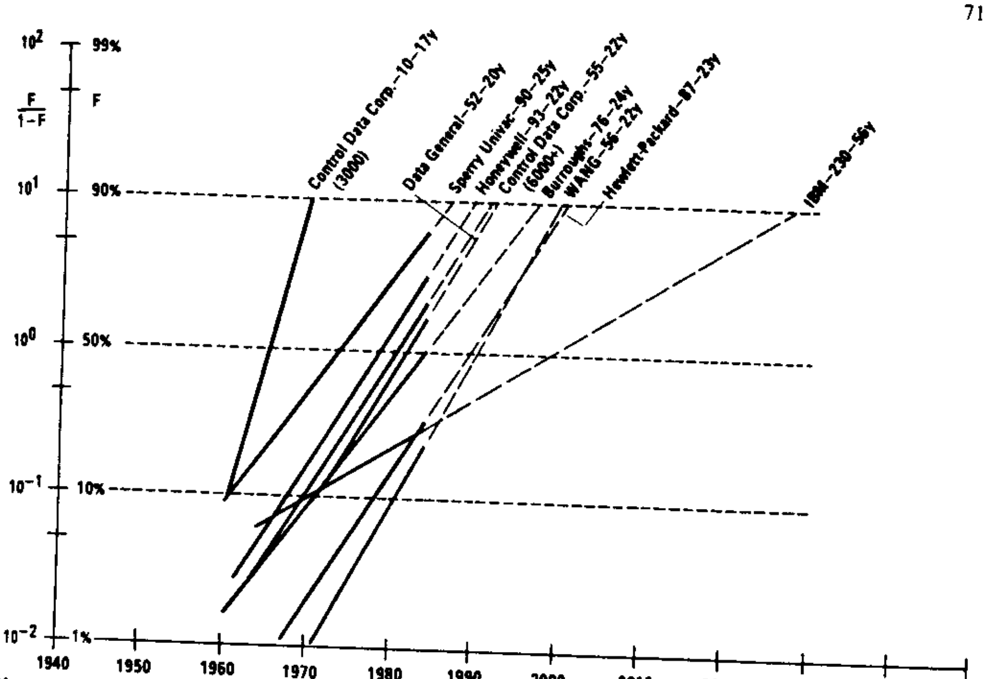

Figure 20. Innovation in computer industry: Mainframe computer models.

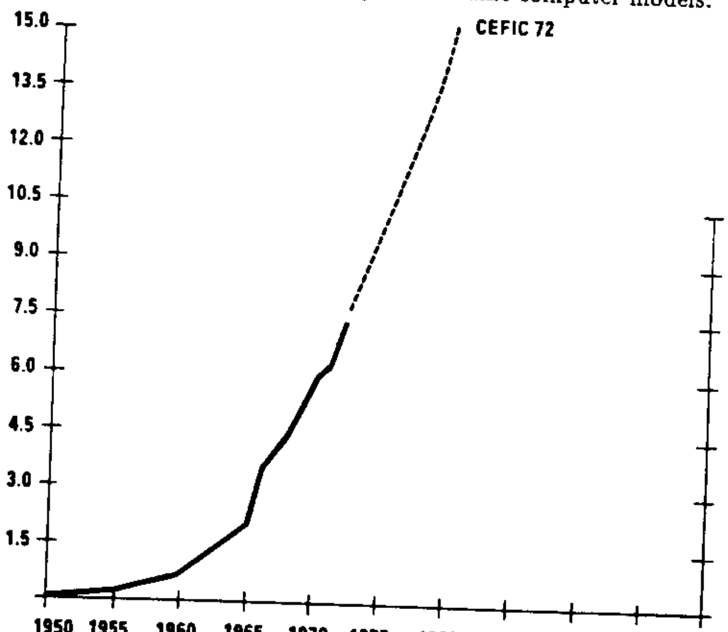

Figure 21. Western Europe ethylene in 1972: Situation and conventional forecast. Source: Grübler (1988).

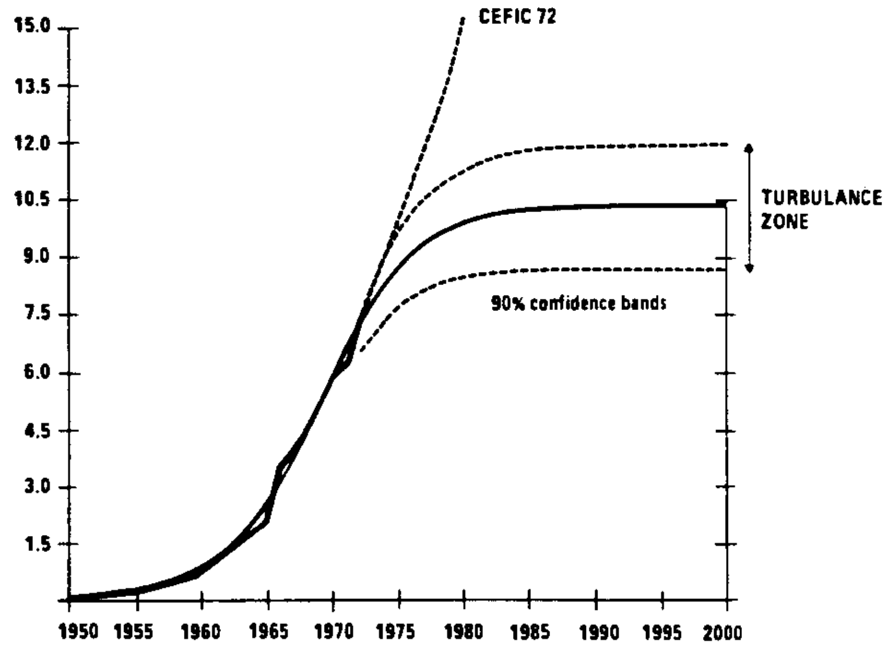

Figure 22. Ethylene in Western Europe in 1972: Anticipating saturation at 10.3 mt. Source: Grübler (1988).

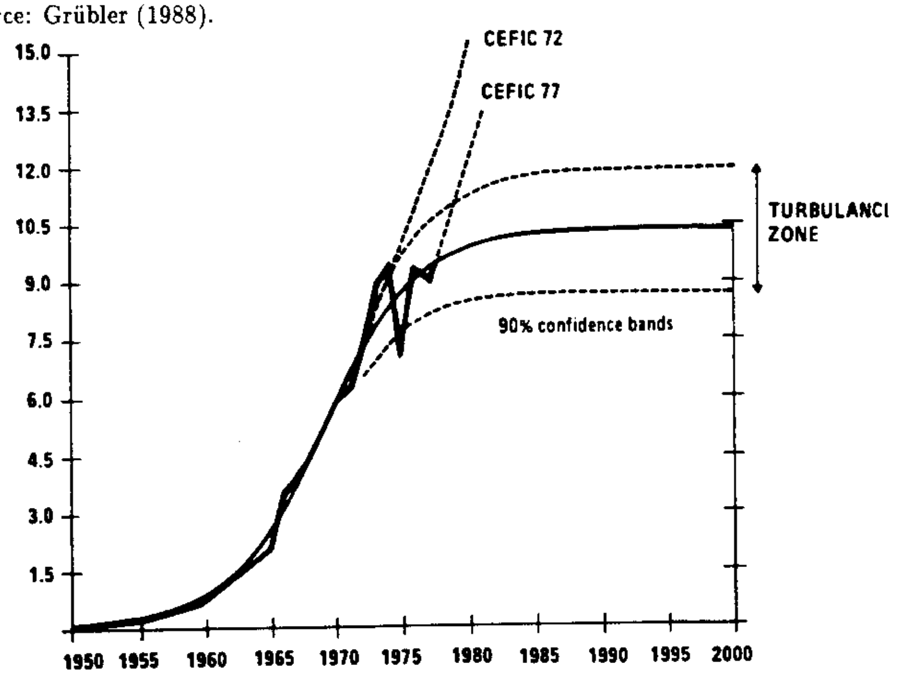

Figure 23. Ethylene in Western Europe in 1977: (1) Oscillation around saturation level; (2) Growth recovery. Source: Grübler (1988).

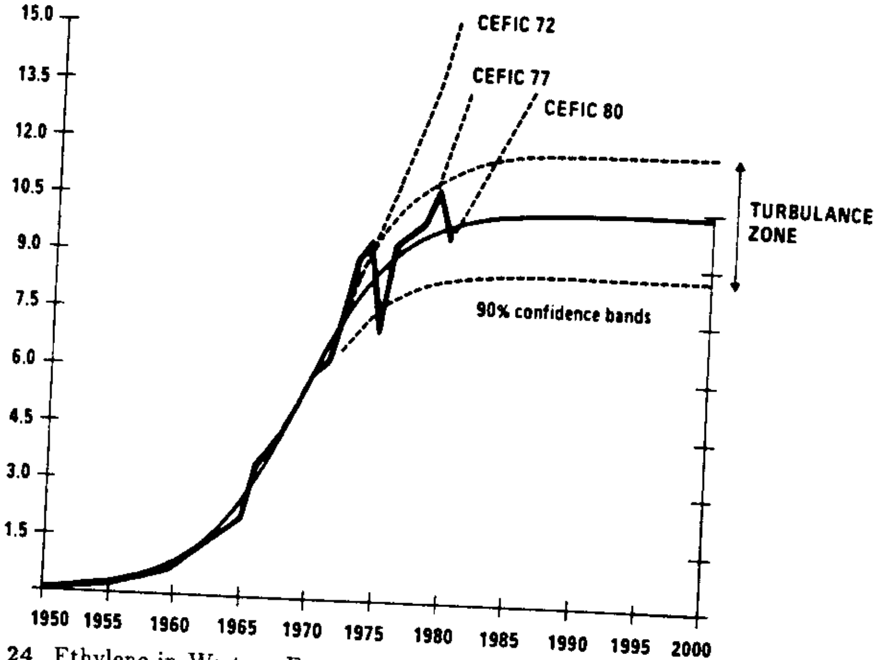

Figure 24. Ethylene in Western Europe in 1980: (1) Oscillation around saturation level; (2) Growth recovery. Source: Grübler (1988).

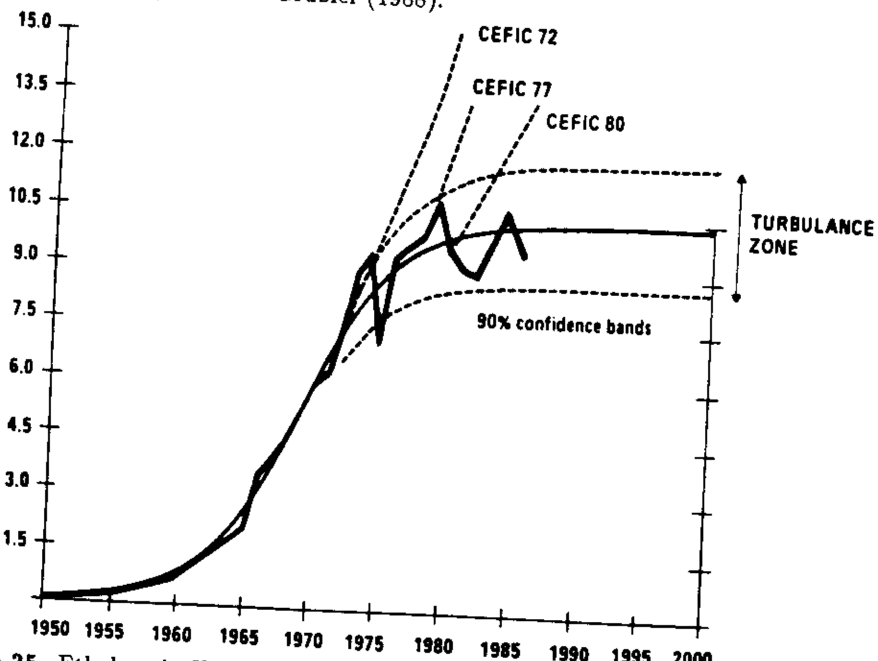

Figure 25. Ethylene in Western Europe in 1988: A history of forecasts. Source: Grübler (1988).

Appendix

The equations for dealing with different cases are reducible to the general Volterra–Lotka equations

$$\frac{dN_i}{dt} = K_i N_i + \beta_i^{-1} \sum_{n}^{j=1} \alpha_{ij} N_i N_j, \tag{A1}$$

where N_i is the number of individuals in species i, and α, β and K are constants. The equation says a species grows (or decays) exponentially, but for the interactions with other species. A general treatment of these equations can be found in Montroll and Goel (1971) and Peschel and Mende (1986). Since closed solutions exist only for the case of one or two competitors, these treatments mainly deal with the general properties of the solutions.

In order to keep the analysis at a physically intuitive level, I use the original treatment of Verhulst (1845) for the population in a *niche* (Malthusian) and that of Haldane (1924) for the competition between two genes of different fitness. For the multiple competition, we have developed a computer package which works perfectly for actual cases (Marchetti and Nakićenović, 1979) but whose identity with the Volterra equations is not fully proven (Nakićenović, 1979).

Most of the results are presented using the co-ordinates for the linear transform of a logistic equation originally introduced by Fisher and Pry (1970).

The Malthusian case. This modeling of the dynamics of population systems started with Verhulst in 1845, who quantified the Malthusian case. A physically very intuitive example is given by a population of bacteria growing in a bottle of broth. Bacteria can be seen as machinery to transform a set of chemicals in the broth into bacteria. The rate of this transformation, *ceteris paribus* (e.g., temperature), can be seen as proportional to the number of bacteria (the transforming machinery) and the concentration of the transformable chemicals.

Since all transformable chemicals will be transformed finally int bacterial bodies, to use homogeneous units one can measure broth chemicals in terms of bacterial bodies. So $N(t)$ is the number of bacteria at time t, and $\overline{N}$ is the amount of transformable chemicals at time 0, before multiplication starts. The Verhulst equation can then be written

$$\frac{dN}{dt} = \alpha N(\overline{N} - N), \tag{A2}$$

whose solution is

$$N(t) = \frac{\overline{N}}{1 - \exp[-(\alpha t + b)]} \ , \tag{A3}$$

with b an integration constant, sometimes written as t_0, i.e., time at time 0; α is a rate constant which we assume to be independent of the size of the population. This means that there is no "proximity feedback". If we normalize to the final size of the system, $\overline{N}$, and explicate the linear expression, we can write equation (A2) in the form suggested by Fisher and Pry (1970):

$$\log \frac{F}{1 - F} = \alpha t + b, \tag{A4}$$

where $F = \frac{N}{\overline{N}}$.

Most of the charts are presented in this form. $\overline{N}$ is often called the *niche*, and the growth of a population is given as the fraction of the niche it fills. It is obvious that this analysis has been made with the assumptions that *there are no competitors.* A single species grows to match the resources ($\overline{N}$) in a Malthusian fashion.

The fitting of empirical data requires calculation of the three parameters $\overline{N}$, a and b, for which there are various recipes of Oliver (1964), Wade-Blackman (1972), and Bossert (1977). The problem is to choose the physically more significant representation and procedure.

I personally prefer to work with the Fisher–Pry transform, because it operates on *ratios* (e.g., of the size of two populations), and ratios seem to me more important than absolute values, both in biology and in social systems.

The calculation of $\overline{N}$ is usually of great interest, especially in economics. However, the value of $\overline{N}$ is very sensitive to the value of the data, i.e., to their errors, especially at the beginning of the growth. The problem of assessing the error on $\overline{N}$ has been studied by Debecker and Modis (1986), using numerical simulation.

The Malthusian logistic must be used with great precaution because it contains implicitly some important hypotheses:

- That there are no competitors in sight.
- That the size of a niche remains constant.
- That the species and its boundary conditions (e.g., temperature for the bacteria) stay the same.

The fact that in multiple competition the starts are always logistic may lead to the presumption that the system is Malthusian. When the transition period starts, there is no way of patching up the logistic fit.

The fact that the niches keep changing, due to the introduction of new technologies, makes this treatment, generally speaking, unfit for dealing with the growth of human populations, a subject where Pearl (1924) first applied logistics. Since the treatment sometimes works and sometimes does not, one can find much faith and disillusionment among demographers.

One-to-one competition. The case was studied by Haldane for the penetration of a mutant or of a variety having some advantage in respect to the pre-existing ones. These cases can be described quantitatively by saying that variety 1 has a reproductive advantage of k over variety 2. Thus, for every generation, the ratio of the number of individuals in the two varieties will be changed by $1/(1\text{-}k)$. If n is the number of generations, starting from $n = 0$, then we can write

$$\frac{N_1}{N_2} = \frac{R_0}{1 - k)^n}, \tag{A5}$$

where $R_0 = \frac{N_1}{N_2}$ at $t = 0$.

If k is small, as it usually is in biology (typically 10^{-3}), we can write

$$\frac{N_1}{N_2} = \frac{R_0}{\exp[kn]}. \tag{A6}$$

We are then formally back to square one, i.e., to the Malthusian case, except for the very favorable fact that we have an initial condition (R_0) instead of a final condition ($\overline{N}$). This means that in *relative terms* the evolution of the system is not sensitive to the size of the niche, a property that is extremely useful for forecasting in multiple-competition cases. Since the generation can be assumed equally spaced, n is actually equivalent to time.

As for the biological case, it is difficult to prove that the "reproductive advantage" remains constant in time, especially when competition lasts for tens of years and the technology of the competitors keeps changing, not to speak of the social and organizational context. But the analysis of hundreds of cases shows that systems behave exactly *as if.*

Multiple competition. Multiple competition is dealt using a computer package originally developed by Nakićenović (1979). A simplified description says that all the competitors start in a logistic mode and phase out in a logistic mode. They undergo a transition from a logistic-in to a logistic-out during which they are calculated as "residuals", i.e., as the difference between the size of the niche and the sum of all the *ins* and *outs.* The details of the rules are to be found in Nakićenović (1979). This package has been used to treat about one hundred empirical cases, all of which showed an excellent match with reality.

An attempt to link this kind of treatment to current views in economics has been made by Peterka (1977).

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